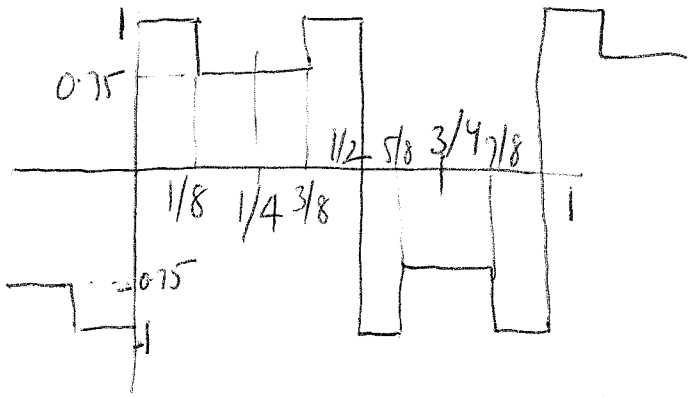
Duration: 50 min Test 2 - Networks and Systems 11/07/07

1. First order low-pass and high-pass circuits with tunable cut-off frequencies are available. Assuming no loading, design appropriate series and/or parallel connections of these circuits to realize (a) bandpass, and (b) bandreject filters with cut off frequencies ω_{c1} and ω_{c2} ($\omega_{c2} > \omega_{c1}$). (40)

2. 

Use the least amount of computation to represent this periodic signal upto the 3rd harmonic. Find its approximate rms value based on the first 3 harmonics. Find its true rms value also. (60)

Solution Test 2 - Fall 07 - EEL3123C

1. (a) A low-pass with cut-off frequency ω_{c2} followed in series by a high-pass with cut-off frequency ω_{c1} makes a bandpass with cut-off frequencies ω_{c1} and ω_{c2}.

(b) A parallel combination of a low pass with cut-off frequency ω_{c1} and a high pass with cut off frequency ω_{c2} will make the appropriate bandreject filter.

2. The function is odd and shows qw symmetry.
$a_0 = 0$, $a_k = 0 \ \forall k$, $b_k = 0$ for even k.

$$b_1 = 8\int_0^{1/4} f(t)\sin(2\pi t)\,dt = 8\left(\int_0^{1/8} \sin(2\pi t)\,dt + \int_{1/8}^{1/4} 0.75\sin(2\pi t)\,dt\right)$$

$$= -\frac{8}{2\pi}\cos 2\pi t\Big|_0^{1/8} - \frac{6}{2\pi}\cos 2\pi t\Big|_{1/8}^{1/4} = \frac{-8}{2\pi}\left(\cos\frac{\pi}{4} - 1\right) - \frac{6}{2\pi}\left(\cos\frac{\pi}{2} - \cos\frac{\pi}{4}\right)$$

$$= -\frac{1}{\pi}\cos\frac{\pi}{4} + \frac{8}{2\pi} = -\frac{1}{\sqrt{2}\pi} + \frac{8}{2\pi} = \frac{1}{\pi}\left(4 - \frac{1}{\sqrt{2}}\right) = \frac{3.292}{\pi} = 1.048 \approx 1.05$$

$$b_3 = 8\int_0^{1/4} f(t)\sin(6\pi t)\,dt = 8\left(\int_0^{1/8} \sin(6\pi t)\,dt + 0.75\int_{1/8}^{1/4} \sin(6\pi t)\,dt\right)$$

$$= \frac{-8}{6\pi}\left(\cos 6\pi t\Big|_0^{1/8}\right) - \frac{6}{6\pi}\left(\cos 6\pi t\Big|_{1/8}^{1/4}\right)$$

$$= -\frac{8}{6\pi}\left(\cos\frac{3\pi}{4} - 1\right) - \frac{6}{6\pi}\left(\cos\frac{3\pi}{2} - \cos\frac{3\pi}{4}\right) = \frac{8}{6\pi} - \frac{1}{3\pi}\cos\frac{3\pi}{4} = 1.5$$

$$f(t) = 1.05\sin(2\pi t) + 0.5\sin(6\pi t)$$

$$A_1 = \sqrt{a_1^2 + b_1^2} = 1.05, \quad A_2 = 0.5$$

$$Frms = \sqrt{\left(\frac{A_1}{2}\right)^2 + \left(\frac{A_2}{2}\right)^2} = \sqrt{\frac{1}{2}\left(1.05^2 + (0.5)^2\right)} = 0.822$$

$$\text{True rms} = \sqrt{\frac{1}{1/4}\int_0^{1/8} f^2\,dt} = \sqrt{4\left(\int_0^{1/8} dt + \frac{9}{16}\int_{1/8}^{1/4} dt\right)}$$

$$= \sqrt{4\cdot\left(\frac{1}{8} + \frac{9}{16}\cdot\frac{1}{8}\right)} = \sqrt{4\cdot\frac{25}{164}\cdot\frac{1}{8}} = \sqrt{\frac{25}{32}} = 0.883$$