UCF Stands For Opportunity

CDA6530: Performance Models of Computers and Networks

Using Simulation for Statistical Analysis and Verification

---- supplement to Random Variable Generation Lecture

SCHOOL OF ELECTRICAL ENGINEERING & COMPUTER SCIENCE

Objective

- We have learned how to generate random variables in simulation, but

- How can we use them?
- What is the purpose for such simulation?

Example: Generate discrete R.V.

- A loaded dice, r.v. X: number shown up
  - P(X=1) = 0.05, P(X=2)= 0.1, P(X=3) =0.15, P(X=4) = 0.18, P(X=5) = 0.22, P(X=6) = 0.3
- Q1: Simulate to generate 100 samples

$$X = \begin{cases} x_0 & \text{if } U < p_0 \\ x_1 & \text{if } p_0 \leq U < p_0 + p_1 \\ \vdots & \\ x_j & \text{if } \sum_{i=0}^{j-1} p_i \leq U < \sum_{i=0}^{j} p_i \\ \vdots & \end{cases}$$

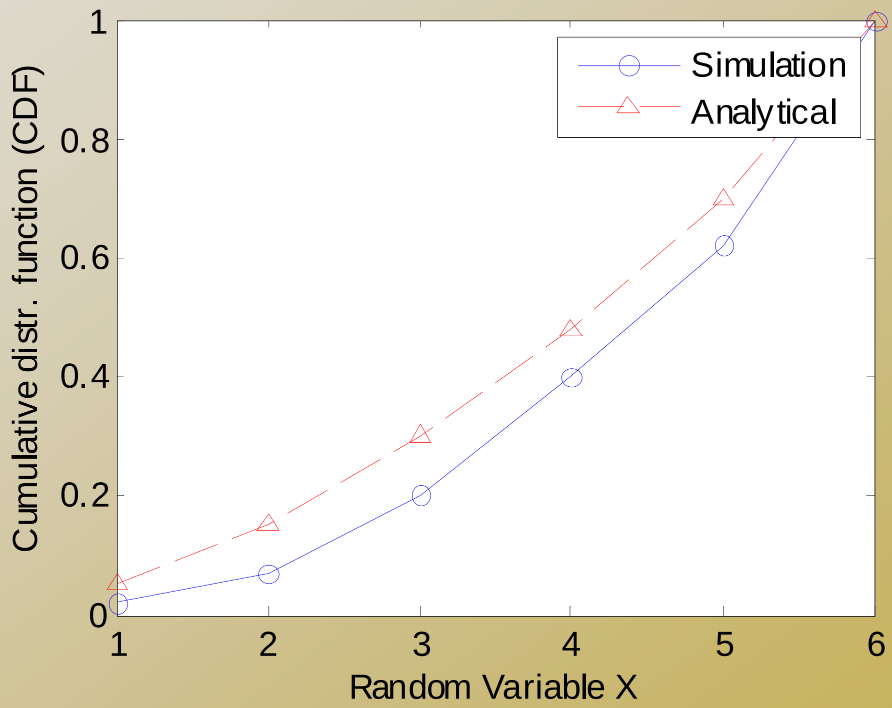
$$X = \begin{cases} 1 & \text{if } U < 0.05 \\ 2 & \text{if } 0.05 \leq U < 0.15 \\ 3 & \text{if } 0.15 \leq U < 0.3 \\ 4 & \text{if } 0.3 \leq U < 0.48 \\ 5 & \text{if } 0.48 \leq U < 0.7 \\ 6 & \text{if } 0.7 \leq U \end{cases}$$

Code in Matlab

Draw CDF (cumulative distr. function)

-
  - Remember $F(x) = P(X \leq x)$
  - For our question, r.v. X has 6 different sample values, thus we could derive from simulation:
    F(1), F(2), F(3), F(4), F(5), F(6)
  - F(6) = 1, so we need to derive the other five
  - F(x) = m/n where
    - n: # of sample values we generate
    - m: # of sample r.v. values $\leq$ x

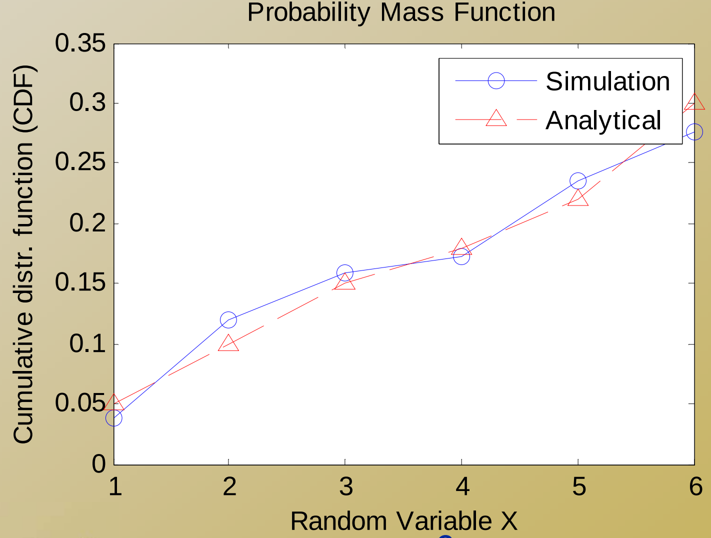
- The CDF curve for our example in one simulation run (compared with analytical results)



Draw PMF (Probability Mass Function)

- Pmf: P(X=1) = 0.05, P(X=2)= 0.1, P(X=3) =0.15, P(X=4) = 0.18, P(X=5) = 0.22, P(X=6) = 0.3

- PMF(x) = m/n
  - n: # of sample values we generate
  - m: # of sample r.v. values = x



Matlab Code

```
% lecture 15, statics-matlab.ppt , Page 3
% generate discrete r.v.
sampleN = 1000;
P = [0.05 0.1 0.15 0.18 0.22 0.3];
Xvalue = [1 2 3 4 5 6];
n =length(P);
SumP = zeros(n, 1);
for i=1:n,
   SumP(i) = sum(P(1:i));
end
U = rand(sampleN,1);
Sample = zeros(sampleN,1);
for i=1:sampleN,
   for j=1:n,
      if U(i)< SumP(j),
         Sample(i) = Xvalue(j);
         break;
      end
   end
end

% draw CDF, Page 5
F = zeros(n,1);
for i = 1:n,
   for j=1:sampleN,
      if Sample(j)<= Xvalue(i),
         F(i) = F(i) + 1;
      end
   end
end
F = F./sampleN;
plot(F, '-ob');
xlabel('Random Variable X'); ylabel('Cumulative distr.
function (CDF)');
hold on;
plot(SumP, '--^r');
legend('Simulation', 'Analytical');
title('CDF');

% draw pmf, Page 6
PMF = zeros(n,1);
for i=1:n,
   for j=1:sampleN,
      if Sample(j) == Xvalue(i),
         PMF(i)= PMF(i)+1;
      end
   end
end
PMF = PMF./sampleN;
figure;
plot(PMF, '-ob');
xlabel('Random Variable X'); ylabel('Cumulative distr.
function (CDF)');
hold on;
plot(P, '--^r');
legend('Simulation', 'Analytical');
title('Probability Mass Function');
```

Continuous R.V.

- Use inverse transform method:
  - One value of U → one r.v. sample
- Normal distr. use the polar method to generate
- How to draw CDF?
  - Problem: r.v. x could be any value
  - Solve: determine x_i points to draw with fixed interval (i=1,2,…)
  - $F(x_i) = P(X \leq x_i) = m/n$
    - n: # of samples generated
    - m: # of sample values $\leq x_i$

Analytical Results

- Use the formula of the distribution to directly calculate F(x_i)
- How to calculate integration in Matlab?
  - Use function quad()

```
Q = quad(@myfun,0,2);
  %where myfun.m is the M-file function:
function y = myfun(x)
    y = 1./(x.^3-2*x-5);
```

Markov Chain Simulation

- Discrete-time Markov Chain
  - Simulate N steps
  - For each step, use random number U to determine which state to jump to
    - Similar to discrete r.v. generation
  - $\pi(i) = m_i/N$
    - N: # of simulated steps
    - m_i: number of steps when the system stays in state i.

Markov Chain Simulation

- Continuous-time Markov Chain
  - Method #1:
    - Determine how long current state lasts
      - Generate exponential distr. r.v. X for the staying time
    - Determine which state the system jumps to
      - Similar to discrete-time Markov Chain jump simulation
  - Method #2:
    - Determine the jumping out time for each jump out link (everyone is expo. Distr.)
    - The staying time is the shortest jumping out time
    - The outgoing state corresponds to this shortest jumping out time
  - Method #2 is more intuitive reasonable

- $\pi(i) = \sum t_k(i) / T$
  - T: overall simulated time
  - $t_k(i)$: the time when the system is in state i for the k-th time