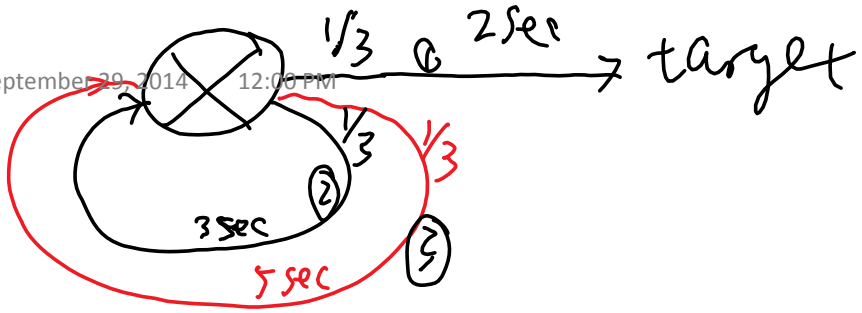


① Model:

r.v. T: time to reach target

r.v. F: path to pick at first step

$\{1, 2, 3\}$

$E[T|F=1] = 2$ sec, $E[T|F=2] = 3 + E[T]$

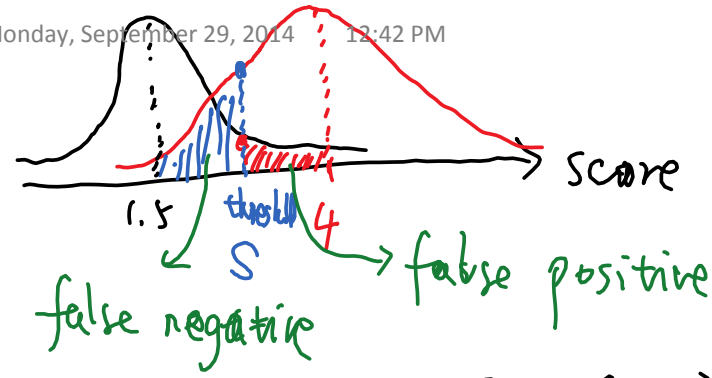
$E[T|F=3] = 5 + E[T]$

② analysis:

$$E[T] = E[T|F=1] \cdot P(F=1) + E[T|F=2] \cdot P(F=2) + E[T|F=3] \cdot P(F=3)$$

$$= \frac{1}{3}\{2 + 3 + E[T] + 5 + E[T]\}$$

$$\Rightarrow E[T] = 2 + 3 + 5 = 10 \text{ sec}$$



Q_1: decide S, such that $P(Y \geq S) = 0.95$

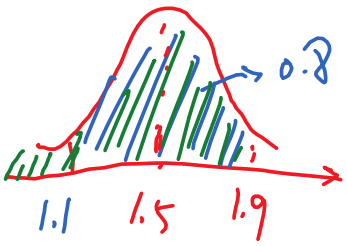
r.v. X: score of normal

Y: score of spam

$Y \sim N(4, 1)$ $\quad X \sim N(1.5, \sigma^2)$

r.v. $Z \sim N(0,1)$ $\quad Z = \frac{X - 1.5}{\sigma} \Rightarrow X = \sigma Z + 1.5$

$P(X \leq 1.5 + 0.4) =$



$P(X \leq 1.9) = 0.8 + 0.1 = 0.9 \Rightarrow P(\sigma Z + 1.5 \leq 1.9) = 0.9$

$P(Z \leq \frac{0.4}{\sigma}) = 0.9$ Check normal table, we know that $\frac{0.4}{\sigma} = 1.3$

$\Rightarrow \sigma = \frac{0.4}{1.3} = 0.3$

Q_2: $P(X > 5.7)$?

$X \sim N(1.5, 0.3^2)$

define $Z = \frac{X - 1.5}{0.3}$ $\quad P(X > 5.7) = P(0.3Z + 1.5 > 5.7) = P(Z > 14)$

$P(Y \geq S) = 0.95$ $\quad Z = \frac{Y - 4}{1}$

$\hookrightarrow P(Z \geq S - 4) = 0.95$ $\quad S - 4 = 1.7$

$S = 5.7$

B: nominal distr. $B(2000, 0.001) \sim X$: # of persons have bad reaction

n p

Q1: $P(X=3) = \binom{2000}{3} p^3 (1-p)^{2000-3}$

$\lambda = n \cdot p = 2$

$X \sim$ Poisson $\lambda = 2$

$P(X=k) = e^{-\lambda} \frac{\lambda^k}{k!}$

$P(X=3) = e^{-2} \cdot \frac{2^3}{3!}$

Q2: $P(X>2) = 1 - P(X \le 2) = 1 - P(X=0) - P(X=1) - P(X=2)$

Q3: $E[Y]$? Y: # of trial before a bad reaction

$Y \sim$ Geometric $p = 0.001$

$E[Y] = \frac{1}{p} = 1000$