

# Optimal Generator Start-Up Strategy for Power System Restoration

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**Abstract**—In an earlier EPRI project, the authors employed a knowledge-based system (KBS) to develop a tool to guide system operators to restart generators after a blackout. In this paper, the authors convert the KBS formulation into a Mixed Integer Quadratically Constrained Program (MIQCP) problem. Taking advantage of the quasiconcave property of the generation ramping curves, an algorithm to solve the generator start-up sequence problem is proposed. The solution method breaks the restoration horizon into intervals and develops the restoration plan by finding the status of each generator at each time interval. Optimality is achieved at each step. The algorithm was tested using the PECO data from the EPRI project.

**Keywords**—Power system restoration, Generation capability, Quasiconcavity

## I. INTRODUCTION

Power system restoration is a complex process that restores the system back to normal operation after an extensive outage of system components. The process involves a large number of generation, transmission and distribution, and load constraints [1-2]. A common approach to this problem is to divide the restoration process into stages (e.g. preparation, system restoration and load restoration) [3]. Nevertheless, one common thread linking each of these stages is the generation availability at each restorative stage for stabilizing the system, establishing the transmission path and restoring load. Following a system blackout, some fossil units may require “cranking” power from outside in order to start the unit. Some units may have time constraints within which the unit can be started successfully or else they have to be off line for an extended period of time before they can be restarted and re-synchronized to the grid. As a result, it is important that, during system restoration, the available system generation capability is maximized. Facing limited black start resources and different system constraints on different generating units,

the maximum available generation can be determined by finding the optimal start-up sequence of all generating units in the system.

A restoration problem can be formulated as a multi-objective and multi-stage nonlinear constrained optimization problem [4]. To develop restoration plans to better assist the operator in making decision during system restoration, several approaches and strategies have been developed. Heuristic methods [5] were used to solve this combinatorial optimization problem, but the computational complexity requires more computational time than practically available during the restoration process. Knowledge based system [6-14] approaches tend to require special software tools of which maintenance and support are expensive and impractical for the power industry. Petri net [15], artificial neural networks [16] and genetic algorithm [17] are novel approaches that mimic system operator actions. However, they may not yield precise solutions at such a crucial time.

Some conventional optimization tools have been proposed to provide more accurate solutions. Among these are based on: mathematical programming [18], dynamic programming [19], mixed-integer programming technique [20] and Lagrangian Relaxation [21]. These optimization technologies require adequate and precise models to achieve the global optimality.

In this paper, the authors proposed a Two-Step optimization algorithm that takes advantage of the “quasiconcave” property of the generation ramping curves to determine the maximum available generation during system restoration. Learning from the experience from developing the KBS restorative tool, the authors formulated the generator start-up sequence into a Mixed Integer Quadratically Constrained Program (MIQCP) problem. The optimal solution is obtained by finding the generator status at each discrete time

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This work was supported by Power Systems Engineering Research Center (PSERC).

interval. Due to the “quasiconcave” property, optimality of the solution at each discrete time period is guaranteed with a highly efficient restoration procedure.

## II. OPTIMAL GENERATOR START-UP STRATEGY

After a system blackout, starting up generators and maintaining maximum system generation capability are important tasks in order to quickly restore the system to a normal state. However, it is a complex combinatorial optimization problem that begins with the optimal use of available black start units to restart other generators while maximizing the overall system generation capability during the restoration period. Note that there are two groups of generating units: black start (BS) generators and non black start (NBS) generators. BS generators, e.g., hydro or combustion turbine units, can be started by itself, while NBS generators, such as steam turbine units, require cranking power from outside.

### A. Generator start-up sequencing problem

**Objective function:** The same objective as the goal driven restoration process in the KBS methodology in [1] is adopted. It is to maximize the overall system generation capability that can be used to restart other NBS units during the restoration period. System generation capability is defined as the total system MW capability minus the start-up requirements.

**Constraints:** NBS generators have different physical characteristics and requirements, i.e., critical minimum & maximum time intervals constraints. If a NBS unit does not start within a critical maximum time interval  $T_{cmax}$ , the unit will not be available until after a considerable time delay. On the other hand, a NBS unit with a critical minimum time interval constraint  $T_{cmin}$ , cannot be restarted until this time interval expires. Moreover, all NBS generators are subjected to start-up power requirement constraints, which they can only be started when the system can supply sufficient cranking power  $P_{start}$ . Instead of setting these constraints as heuristic rules, the generator start-up sequencing problem is formulated as the following optimization problem:

Max Overall System Generation Capability  
subject to Critical Minimum & Maximum Time Intervals  
Start-Up Power Requirements

Then define the following sets:

- **ASG**: set of all already started generators;
- **BSG**: set of all BS generators;
- **NBSG**: set of all NBS generators;
- **NBSGMIN**: set of NBS generators with constraint of  $T_{cmin}$ ;
- **NBSGMAX**: set of NBS generators with constraint of  $T_{cmax}$ .

### B. “Two-Step” generation capability curve

The MW capability of each BS or NBS generator  $P_{igen}$  can be expressed by the area between its generation capability curve and the time horizon, as shown in Fig. 1.

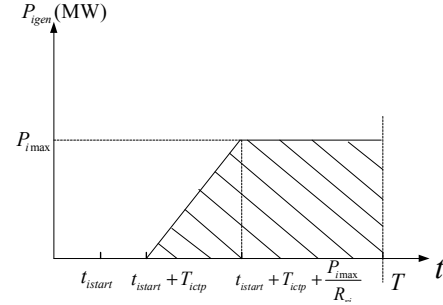


Figure 1. Generation capability curve

where  $P_{max}$  is the maximum generator active power output,  $t_{start}$  is generator starting time,  $t_{icp}$  is cranking time for generators to begin to ramp up and parallel with the system,  $R_r$  is generator ramping rate, and  $T$  is the total restoration time.

**Definition of quasiconcavity:** A function  $f$  is quasiconcave if and only if for any  $x, y \in \text{dom } f$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\} \quad (1)$$

In other words, the value of  $f$  over the interval between  $x$  and  $y$  is not smaller than  $\min\{f(x), f(y)\}$ .

With the above definition, one can obtain the following lemma:

**Lemma 1:** The generation capability function is quasiconcave.

*Proof:* See the Appendix.

Convex optimization is concerned with minimizing convex functions or maximizing concave functions. Optimality cannot be guaranteed without the property of convexity or concavity. Due to the quasiconcavity property, one cannot directly use the general convexity-based or concavity-based optimization method for developing solutions.

Therefore, a “Two-Step” method is proposed to solve the quasiconcave optimization problem. For each generator, the generation capability curve is divided into two segments. One segment  $P_{igen1}$  is from the origin to the “corner” point where the generator begins to ramp up, as shown by the red line in Fig. 2. The other segment  $P_{igen2}$  is from the corner point to point when all generators have been started, as shown by the blue line in Fig. 2. The quasiconcave function is converted into two concave functions. Then time horizon is divided into several time periods, and in each time period, generators using either first or second segment of generation capability curves. The quasiconcave optimization problem is converted into concave optimization problem, which optimality is guaranteed in each time period.

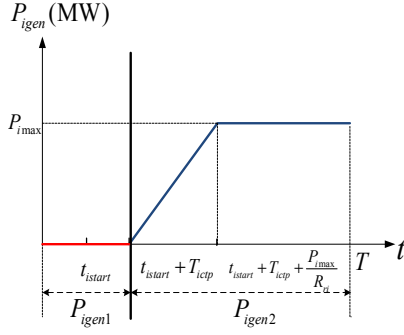


Figure 2. Generation capability curve

### C. Algorithm

Start solving the optimization problem with all generators using the first segment of generation capability function  $P_{igen1}(t)$ . The restoration time  $T$  at which all NBS generators (excluding nuclear generators that usually require restart time greater than the largest critical minimum time of all generators) have been restored, is discretized into  $N_T$  equal time slots. Beginning at  $t = 1$ , the optimization problem is solved and the solution is recorded. Then at  $t = 2$ , the problem is solved again to update the solution. This iteration continues until  $t = N_T$  by advancing the time interval according to the following criteria:

- 1) If generation capability function of every generator  $\in \mathbf{ASG}$  has been updated from  $P_{igen1}(t)$  to  $P_{igen2}(t)$ , set  $t = t + 1$ ;
- 2) If every generator  $\in \mathbf{NBSGMAX}$  have been started, set  $t = \min \{T_{icmin}, i \in \mathbf{NBSMIN}\}$ ;
- 3) If all generators have been started, set  $t = T$ ;
- 4) Otherwise, set  $t = \min \{t_{istart} + T_{ictp}, i \in \mathbf{ASG}\}$ .

Then in the next iteration, if any generator reaches its maximum capability, update the generation capability function from  $P_{igen1}(t)$  to  $P_{igen2}(t)$ . At this time, some generators are in their first segments of the capability curves and others are in the second segments. During the process, if any new generator was started, add it to the set  $\mathbf{ASG}$ . Then the problem can be solved each time period by time period until all generators have been started. The number of total time periods is different for each individual case.

## III. PROBLEM FORMULATION

### A. Objective Function

The objective function can be written as

$$\max \sum_{t=1}^{N_T} \sum_{i=1}^N [P_{igen}(t) - u_i^t (1 - u_i^{t-1}) P_{istart}(t)] \quad (2)$$

where  $N$  is the number of total generation units, binary decision variable  $u_i^t$  is the status of NBS generator at each time slot, which  $u_i^t = 1$  means  $i_{th}$  generator is on at time  $t$ , and  $u_i^t = 0$

means  $i_{th}$  generator is off. It is assumed that all BS generators are started at the beginning of restoration.

### B. Constraints

Critical Time Constraints: Generators with constraints of  $T_{cmax}$  or  $T_{cmin}$  should satisfy the following inequalities:

$$\left. \begin{aligned} t_{istart} &\geq T_{icmin}, & i \in \mathbf{NBSGMIN} \\ t_{istart} &\leq T_{icmax}, & i \in \mathbf{NBSGMAX} \end{aligned} \right\} \quad (3)$$

Start-Up MW Requirements Constraints: NBS generators can only be started when the system can supply sufficient cranking power:

$$\sum_{i=1}^N [P_{igen}(t) - u_i^t (1 - u_i^{t-1}) P_{istart}(t)] \geq 0, t = 1, \dots, N_T \quad (4)$$

Generator capability function  $P_{igen}(t)$  can be expressed as:

$$P_{igen}(t) = P_{igen1}(t) + P_{igen2}(t) \quad (5)$$

where,

$$P_{igen1}(t) = 0 \quad 0 \leq t < t_{istart} + T_{ictp} \quad (6)$$

$$P_{igen2}(t) = R_{ri} (t - t_{istart} - T_{ictp}) \quad (7)$$

$$P_{igen2}(t) \leq P_{imax} \quad (8)$$

Generator Status Constraints: It is assumed that once generator was restarted, it will not be tripped offline again, which is guaranteed by the following inequalities:

$$u_i^{t-1} \leq u_i^t, \quad i = 1, \dots, N, t = 2, \dots, N_T \quad (9)$$

Then the generator start-up sequencing problem can be formulated as a Mixed Integer Quadratically Constrained Program (MIQCP):

$$\begin{aligned} \max & \sum_{t=1}^{N_T} \sum_{i=1}^N [(P_{igen1}(t) + P_{igen2}(t)) - u_i^t (1 - u_i^{t-1}) P_{istart}] \\ \text{s.t.} & \quad t_{istart} \geq T_{icmin}, \quad i \in \mathbf{NBSGMIN} \\ & \quad t_{istart} \leq T_{icmax}, \quad i \in \mathbf{NBSGMAX} \\ & \quad \sum_{i=1}^N [(P_{igen1}(t) + P_{igen2}(t)) - u_i^t (1 - u_i^{t-1}) P_{istart}] \geq 0 \quad (10) \\ & \quad P_{igen1}(t) = 0 \quad 0 \leq t < t_{istart} + T_{ictp} \\ & \quad P_{igen2}(t) = R_{ri} (t - t_{istart} - T_{ictp}) \\ & \quad P_{igen2}(t) \leq P_{imax} \\ & \quad u_i^{t-1} \leq u_i^t, \quad t = 2, \dots, N_T \\ & \quad i = 1, \dots, N, \quad t = 1, \dots, N_T \\ & \quad u_i^t \in \{0, 1\}, \quad t_{istart} \text{ integer} \end{aligned}$$

### C. Flow chart of the algorithm

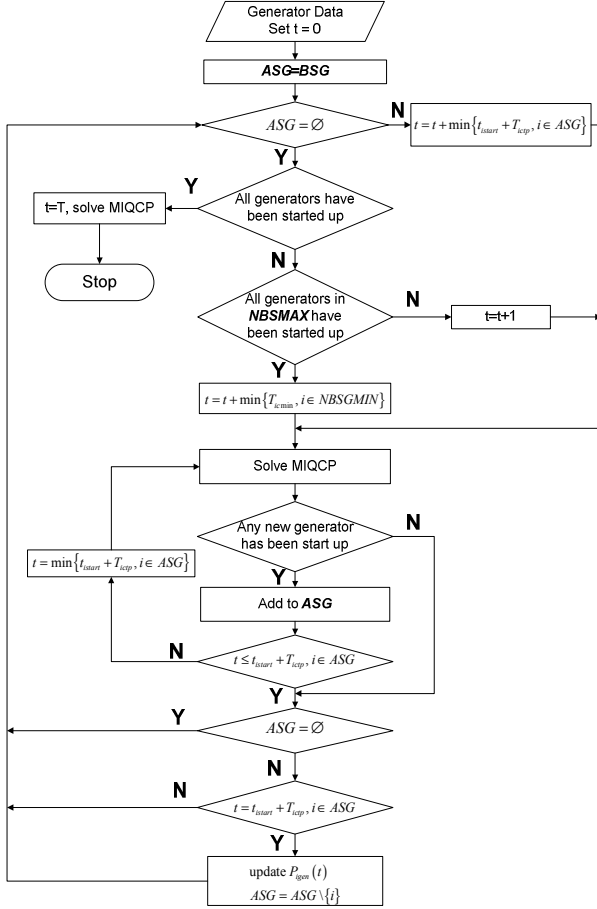


Figure 3. Flow chart of “Two-Step” algorithm

## IV. NUMERICAL RESULTS

In this research, the software tool of ILOG CPLEX is used to solve the proposed MIQCP. CPLEX provides Simplex Optimizer and Barrier Optimizer to solve the problem with continuous variables, and Mixed Integer Optimizer to solve the problem with discrete variables. ILOG CPLEX Mixed Integer Optimizer includes sophisticated mixed integer preprocessing routines, cutting-plane strategies and feasibility heuristics. The default settings of MIP models are used with a general and robust branch & cut algorithm.

### A. Four-generator system

A four-generator system with fictitious data is studied to illustrate the algorithm. Table I gives the generator characteristics.

TABLE I. DATA OF GENERATOR CHARACTERISTICS

$i$	$T_{ctp}$	$T_{cmin}$	$T_{cmax}$	$R_r$ (MW/ per unit time)	$P_{start}$ (MW)	$P_{max}$ (MW)
1	2	N/A	5	2	1	8
2	1	5	N/A	4	1	12
3	2	N/A	4	4	2	20
4	1	N/A	N/A	1	N/A	3

In this system, there are 3 NBS generators and 1 BS generator. Among the 3 NBS generators, 2 units have  $T_{cmax}$  and 1 unit has  $T_{cmin}$ . The total restoration time is set to be 12 time units. The optimal starting time for all generating units is obtained after 5 iterations by applying the proposed method, as shown in Table II.

TABLE II. GENERATOR STARTING TIMES

Unit	$t_{start}$ (per unit time)
1	2
2	5
3	4
4	0

Table III gives the generator status of the optimal solution:

TABLE III. GENERATOR STATUS FOR THE OPTIMAL SOLUTION

Time	NBS Generator				System Generation Capability (MW)
	$i=1$	$i=2$	$i=3$	$i=4$	
$t=0$	0	0	0	1	0
$t=1$	0	0	0	1	0
$t=2$	1	0	0	1	0
$t=4$	1	0	1	1	0
$t=6$	1	1	1	1	3
$t=12$	1	1	1	1	39

Fig. 4 shows the time instants where generators change to the respective second segment of the capability function. The red line is system total generation capability curve.

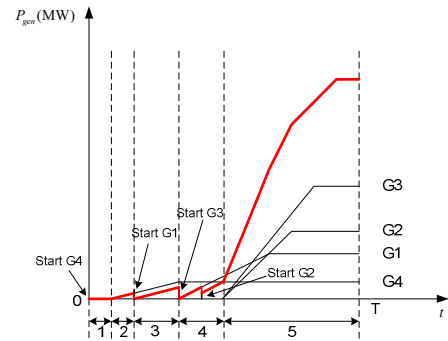


Figure 4. Two steps of generation capability curve

The following stages summarize how the restoration process progresses:

- 1) In the beginning, BS generator G4 is started at  $t=0$ , and add it to  $ASG$ .
- 2) In time period 1, according to criterion (4), set  $t=t_{start}+t_{ctp}=1$ , and solve the problem. Update generation capability function of G4 to  $P_{4gen2}(t)$ .
- 3) In time period 2, by criterion (1), set  $t=1+1=2$ , and solve the problem. It is shown that NBS generator G1 is started, and add it to  $ASG$ .
- 4) Then in time period 3, set  $t=t_{start}+t_{ctp}=4$  by criterion (4), and solve the problem again. It is shown that NBS generator G3 is started, and add it to  $ASG$ . Update the generation capability function of G1 to  $P_{1gen2}(t)$ .

- 5) In time period 4, set  $t=t_{3start}+t_{3ctp}=6$  according to criterion (4) and solve the problem. NBS generator G2 is started, and adds it to *ASG*. Update the generation capability function of G3 to  $P_{3gen2}(t)$ .
- 6) In time period 4, according to criterion (3), set  $t=T=12$ , and solve the problem.

As shown in Fig. 4, there are a total of five time periods to calculate the optimal solution for four-generator system.

### B. PECO system

The proposed algorithm is applied to the generators in the PECO system. For simplicity, units at the same station with similar characteristics are aggregated into one [1]. Table IV gives the generator characteristic data.

TABLE IV. DATA OF GENERATOR CHARACTERISTICS

Unit	Type	$T_{ctp}$ (hr)	$T_{cmin}$ (hr)	$T_{cmax}$ (hr)	$R_r$ (MW/hr)	$P_{start}$ (MW)	$P_{max}$ (MW)
Chester_4-6	CT	N/A	N/A	N/A	120	N/A	39
Conowingo_1-11	Hydro	N/A	N/A	N/A	384	N/A	560
Cromby_1-2	Steam	1:40	N/A	N/A	148	8	345
Croydon_1	CT	0:30	5:00	N/A	120	6	384
Delaware_9-12	CT	N/A	N/A	N/A	162	N/A	56
Eddystone_1-4	Steam	1:40	3:20	N/A	157	12	1341
Eddystone_10-40	CT	N/A	N/A	N/A	168	N/A	60
Falls_1-3	CT	N/A	N/A	N/A	135	N/A	51
Moser_1	CT	N/A	N/A	N/A	90	N/A	51
Muddy Run_1-8	Hydro	0:30	N/A	N/A	246	13.2	1072
Richmond_91_92	CT	N/A	N/A	N/A	288	N/A	96
Schuylkill_1	Steam	2:00	N/A	2:30	135	2.7	166
Schuylkill_10-11	CT	N/A	N/A	N/A	84	N/A	30
Southwark_3-6	CT	N/A	N/A	N/A	156	N/A	52
CCU1	CC	2:40	N/A	3:20	108	5	500
CCU2	CC	2:00	2:30	N/A	162	7.5	500

In this system, there are 7 NBS generators and 9 BS generators. Among 7 NBS generators, 2 units have  $T_{cmax}$  and 3 other units have  $T_{cmin}$ . The total restoration time is set to be 15 hrs, which is divided into 90 time slots with a 10 min length for each time slot.

After a blackout, the optimal starting time for all generating units is obtained after 9 iterations by applying the proposed algorithm. The results are shown in Table V.

TABLE V. GENERATOR STARTING TIMES

$i$	Unit	$t_{start}$ (hr)
3	Cromby_1-2	0:10
4	Croydon_1	5:00
6	Eddystone_1-4	3:20
10	Muddy Run_1-8	0:10
12	Schuylkill_1	0:10
15	CCU1	0:10
16	CCU2	2:30

Table VI gives the generator status for the optimal solution:

TABLE VI. GENERATOR STATUS FOR OPTIMAL SOLUTION

Time	NBS Generator						System Generation Capability (MW)
	$i=3$	$i=4$	$i=6$	$i=10$	$i=12$	$i=15$	
$t=0$	0	0	0	0	0	0	0
$t=1$	<u>1</u>	0	0	<u>1</u>	<u>1</u>	<u>1</u>	0
$t=4$	1	0	0	1	1	1	0
$t=11$	1	0	0	1	1	1	0
$t=13$	1	0	0	1	1	1	0
$t=17$	1	0	0	1	1	1	1
$t=27$	1	0	1	1	1	1	1
$t=30$	1	<u>1</u>	1	1	1	1	1
$t=33$	1	1	1	1	1	1	1
$t=90$	1	1	1	1	1	1	1

Fig. 5 shows the time instants where generators change to the respective second segment of the capability function. The red line is system total generation capability curve.

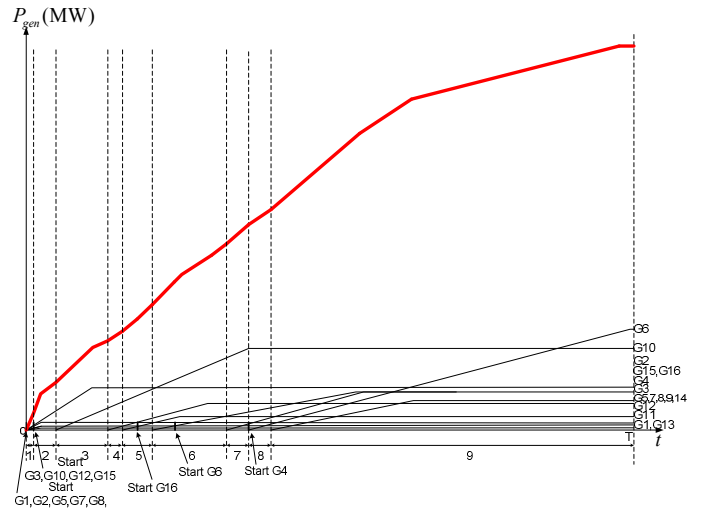


Figure 5. Two steps of generation capability curve

The following is a summary of the restoration process:

- 1) In the beginning, BS generators G1, G2, G5, G7, G8, G9, G11, G13 and G14 are started up  $t=0$ , and add them to *ASG*.
- 2) In time period 1, since none of BS generators have the characteristic of  $T_{ctp}$ , according to criterion (1), set  $t=0+1=1$ , and solve the problem. It is shown that NBS generator G3, G10, G12 and G15 are started, and add them to *ASG*.
- 3) In time period 2, by criterion (4), set  $t= t_{10start}+t_{10ctp}=4$ , and solve the problem. Update the generation capability curve of G10 to  $P_{10gen2}(t)$ .
- 4) Then in time period 3, set  $t= t_{3start}+t_{3ctp}=11$  by criterion (4), and solve the problem again. Update the generation capability curve of G3 to  $P_{3gen2}(t)$ .
- 5) In time period 4, set  $t= t_{12start}+t_{12ctp}=13$  according to criterion (4), and solve the problem again. Update the generation capability curve of G12 to  $P_{12gen2}(t)$ .

- 6) In time period 5, according to criterion (4), set  $t = t_{15start} + t_{15ctp} = 17$ , and solve the problem again. It is shown that NBS generator G16 is started at  $t = 15$ , and add it to *ASG*. Update the generation capability curve of G15 to  $P_{15gen2}(t)$ .
- 7) In time period 6, by criterion (4), set  $t = t_{16start} + t_{16ctp} = 27$ , and solve the problem again. It is shown that NBS generator G6 is started at  $t = 20$ , and add it to *ASG*. Update the generation capability curve of G16 to  $P_{16gen2}(t)$ .
- 8) In time period 7, set  $t = t_{6start} + t_{6ctp} = 30$  by criterion (4), and solve the problem again. It is shown that NBS generator G4 is started at  $t = 30$ , and add it to *ASG*. Update the generation capability curve of G6 to  $P_{6gen2}(t)$ .
- 9) In time period 8, by criterion (4), set  $t = t_{4start} + t_{4ctp} = 33$ , and solve the problem. Update the generation capability curve of G4 to  $P_{4gen2}(t)$ .
- 10) In time period 9, according criterion (3), set  $t = T = 90$ , and solve the problem.

There are total nine time periods, as shown in Fig. 5, to solve the optimization problem for PECO system.

## V. CONCLUSION

In this paper, the authors have successfully formulated the previously developed KBS based restoration tool into a MIQCP optimization problem for determining an optimal generator start-up strategy for power system restoration following a blackout. Incorporating the proposed ‘‘Two-Step’’ algorithm to take advantage of the quasiconcave property of generation capability curve, the optimization problem can be solved with available convexity-based optimization tools. The numerical results demonstrate the accuracy of the models and effectiveness of the algorithm. While the solution provides system operators an optimal start-up sequence of the generators at the start of the system restoration, system operators need to identify transmission paths and pick up critical loads as the restoration effort continues. Since these needs are company and system specific, the path finding and load pickup process are not a part of this paper and will be considered as this research continues.

The contribution of this research is in the formulation of the generator starting sequence problem into a precise mathematical programming problem. Compared to the empirical solutions based on heuristic methods or other knowledge-based approaches, true optimal solutions are obtained at each time step. Added to that, the specific formulation does not depend on highly special software tools, the optimization formulation appears to be free of special maintenance and support requirements and is more practical and attractive for the long term development of a decision support tool.

## VI. DISCLAIMER

The opinions expressed in this paper are the opinions of the authors and do not reflect the opinions of the authors’ institution, company or its affiliates.

## A. Proof of Lemma 1

First, divide  $\text{dom } f$  into three consecutive sets, and  $\text{dom } f = S_1 \cup S_2 \cup S_3$ :

$$S_1 = \{t : 0 \leq t < t_{start} + t_{ctp}\},$$

$$S_2 = \{t : t_{start} + t_{ctp} \leq t < t_{start} + t_{ctp} + P_{max} / R_r\},$$

$$S_3 = \{t : t_{start} + t_{ctp} + P_{max} / R_r \leq t \leq T\},$$

Then, consider all possible cases:

- 1) If for any  $x, y \in S_1$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) = f(x) = f(y) \geq \min\{f(x), f(y)\}$$

- 2) If for any  $x \in S_1, y \in S_2$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq f(x)$$

Since  $f(y) \geq f(x)$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

- 3) If for any  $x \in S_1, y \in S_3$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq f(x)$$

Since  $f(y) \geq f(x)$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

- 4) If for any  $x \in S_2, y \in S_1$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq f(y)$$

Since  $f(x) \geq f(y)$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

- 5) If for any  $x, y \in S_2$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) = f(x) = f(y) \geq \min\{f(x), f(y)\}$$

- 6) If for any  $x \in S_2, y \in S_3$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq f(x)$$

Since  $f(y) \geq f(x)$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

- 7) If for any  $x \in S_3, y \in S_1$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq f(y)$$

Since  $f(x) \geq f(y)$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

8) If for any  $x \in S_3, y \in S_2$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq f(y)$$

Since  $f(x) \geq f(y)$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

9) If for any  $x, y \in S_3$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) = f(x) = f(y) \geq \min\{f(x), f(y)\}$$

From all above, for any  $x, y \in \text{dom } f$  and  $0 \leq \theta \leq 1$ ,

$$f(\theta x + (1-\theta)y) \geq \min\{f(x), f(y)\}$$

Therefore, the generation capability function is quasiconcave.

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