

## Optimal Transmission Path Search in Power System Restoration

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### Abstract

Power system restoration is important for system planning and operation. It enables a grid to be restored from major outages efficiently and safely. The restoration process can be divided to three stages: starting generators, establishing transmission network, and restoring load services. After determining the start-up sequence of generating units, it is critical to find the shortest transmission path to energize transmission network using cranking power from available generators. This paper identified a set of logical constraints for transmission network energization, and formulated the transmission path search as a Mixed Integer Linear Programming (MILP) problem. These constraints could be easily incorporated with other restoration actions such as starting up generating units. Simulation results based on IEEE 39-bus test case demonstrate that the proposed MILP-based transmission path search algorithm is highly efficient and the restoration duration is significantly reduced.

### Introduction

Resilience and efficient recovery are critical and desirable features for electric power systems [1]. Smart grid technologies are expected to enable a grid to be restored from major outages efficiently and safely. As a result, power system restoration is increasingly important for system planning and operation. Following a partial or complete system blackout, the restoration process returns the system back to a normal operating condition by assessing system conditions, starting blackstart (BS) units, establishing the transmission paths to crank non-blackstart (NBS) generating units, picking up the necessary loads to stabilize the power system, and synchronizing the electrical islands [2-3]. Generating units will be started first in an optimal sequence so that the overall system generation capability will be maximized [4]. Then the shortest transmission path needs to be identified to transfer the cranking power and energize the transmission network. After establishing generation and transmission systems, load service will be restored. Current restoration guidelines follow this sequential process.

A comprehensive strategy to facilitate system restoration has been proposed to develop computational modules for the generation, transmission and distribution subsystems [4]. The four primary modules are Generation Capability Optimization Module (GCOM), Transmission Path Search Module (TPSM), Load Restoration Module (LRM), and Constraint Checking Module (CCM), as shown in Fig. 1.

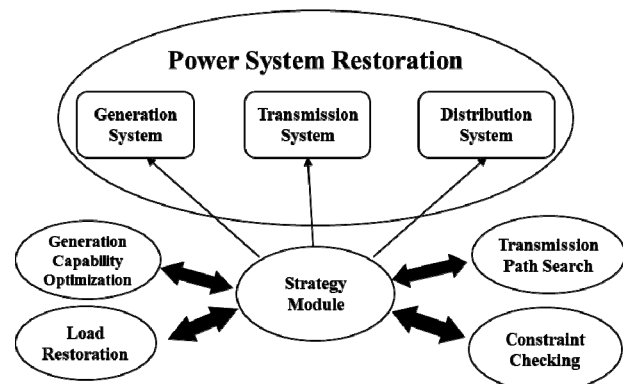


Fig. 1 Power system restoration strategy

These optimization modules interact with each other to develop a power system restoration plan that incorporates generation, transmission, distribution and load constraints. Specifically, GCOM first provides an optimal generator starting sequence, and TPSM identifies the paths for the cranking sequence from GCOM and energizes the transmission network. Then LRM provides the load pickup sequence to maintain the system stability and minimize the unserved load. The sequences from GCOM and LRM, as well as the transmission paths from TPSM, are checked by CCM using power system simulation software tools to ensure that various constraints are met. They interact with each other by receiving the input from and passing the output to other modules. Eventually, this will lead to the successful restoration of a power system.

This complex system restoration problem can be formulated as a combinatorial optimization problem with

practical time-varying constraints. Advanced optimization techniques are needed to provide efficient solutions.

This paper is focused on the transmission path search. It is an important extension to our prior work in [4], which used a simplistic method to configure the transmission network after an optimal generation start-up sequence determined. In literature, several approaches and analytical tools have been proposed to find the shortest transmission path, such as, expert system [5], artificial neural networks [6], and power transfer distribution factor (PTDF)-based approach [7]. These methods integrate both dispatchers' knowledge and computational algorithms for system analysis.

In contrast to existing methods, the proposed approach identified the connection between bus and line status when energizing transmission lines. Then a set of logical constraints are formulated as an MILP problem. The shortest transmission path can be successfully obtained. Moreover, these constraints could be easily integrated with other restoration actions including starting up generating units. Unlike the sequential process, this integrated restoration strategy will determine generation and transmission status simultaneously. Thus, the restoration duration will be significantly reduced.

## Problem Formulation

After optimal generator start-up sequence is identified, transmission lines must be available to deliver cranking power to NBS units or large motor loads, and transformer units, including step-up transformers of BS units and steam turbine units, and auxiliary transformers serving motor control centers at the steam plant [8]. By assigning each bus or line one binary decision variable, denoted by  $u_{busm}^t$  or  $u_{linemn}^t$ . The variable represents its status, which 1 represents energized and 0 represents de-energized. Then the optimal transmission path search problem can be formulated as a MILP problem to find the status of each bus or line at each time. The MILP formulation of generator start-up sequence can be referred in [4]. The detailed formulations of MILP-based transmission path search are shown as following.

**Objective Function:** The objective is to maximize the overall system MW generation capability during a specified system restoration period. The system generation capability is defined as the sum of MW generation capabilities over all units in the power system minus the start-up power requirements, as referred in [4].

$$\begin{aligned} \text{Max}_{t_{jstart}} \left\{ \sum_{i \in \Omega_{ALLU}} \left[ (P_{imax})^2 / (2Rr_i) + P_{imax} (T - T_{ictp} - P_{imax} / Rr_i) \right] \right. \\ \left. - \sum_{j \in \Omega_{NBSU}} P_{jstart} T \right\} - \left( \sum_{i \in \Omega_{ALLU}} P_{imax} t_{istart} - \sum_{j \in \Omega_{NBSU}} P_{jstart} t_{jstart} \right) \end{aligned} \quad (1)$$

where,  $t_{istart}$  is the starting time of NBSU  $i$ ,  $\Omega_{ALLU} = 1 \dots N_{ALLU}$  is the set of all generators number,  $N_{ALLU}$  is total generator number,  $P_{imax}$  is the maximum generation output of generator  $i$ ,  $Rr_i$  is the ramping rate of generator  $i$ ,  $T_{ictp}$  is the cranking time for generator  $i$  to begin to ramp up and parallel with system,  $P_{istart}$  is the start-up power requirement of NBSU  $i$ ,  $\Omega_{BSU} = 1 \dots N_{BSU}$  is the set of BSU number,  $N_{BSU}$  is the total BSU number,  $\Omega_{NBSU} = 1 \dots N_{NBSU}$  is the set of NBSU number,  $N_{NBSU}$  is the total NBSU number, and  $T$  is total restoration time.

The first component (in braces) is constant, then (1) is equivalent to:

$$\text{Min}_{t_{jstart}} \sum_{j \in \Omega_{NBSU}} (P_{jmax} - P_{jstart}) t_{jstart} \quad (2)$$

### Constraints:

A set of logical constraints for transmission network energization has been identified and converted to the MILP problem. Assume the energization for each component takes 1 per unit (p.u.) time. The example of Bus  $m$  and Bus  $n$  are connected through Line  $mn$  is shown in Fig. 2 for the illustration of logical constraints.

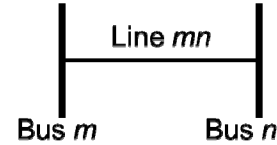


Fig. 2 Illustration of logical constraints for transmission path search

1. If both connected buses are de-energized at  $t$ , then the line is de-energized at  $t$ . For example in Figure 2, if Bus  $m$  and Bus  $n$  are de-energized at  $t$ , there is no power from either end to energize Line  $mn$ , then Line  $mn$  is also de-energized at  $t$ . If either bus is energized, the line can be energized or de-energized, i.e., the line status is free.

$$\begin{aligned} -Mu_{busm}^t \leq u_{linemn}^t \leq Mu_{busm}^t \\ -Mu_{busn}^t \leq u_{linemn}^t \leq Mu_{busn}^t \end{aligned} \quad (3)$$

$t \in \Omega_t, mn \in \Omega_{line}, m \& n \in \Omega_{bus}$

where,  $M$  is the arbitrarily large number,  $\Omega_{bus} = 1 \dots N_{bus}$  is the set of Bus number,  $N_{bus}$  is the total bus number,  $\Omega_{line} = 1, \dots, N_{line}$  is the set of Line

number,  $N_{line}$  is the total line number,  $\Omega_t = 1, \dots, T$  is the set of time.

2. If both connected buses are de-energized at  $t$ , then the line is de-energized at  $t+1$ ; if the line is energized at  $t+1$ , then at least one of connected buses is energized at  $t$ . This constraint clarifies the time of energization (assuming 1 p.u. time). For example in Fig. 2, if Bus  $m$  and Bus  $n$  are de-energized at  $t$ , Line  $mn$  is de-energized at  $t$  by Constraint 1, then Line  $mn$  is also de-energized at  $t+1$ , because at least 1 p.u. time is need to energize buses first. Also, if Line  $mn$  is energized at  $t+1$ , then at least one of the end buses is energized at  $t$ .

$$-M(u_{busm}^t + u_{busn}^t) \leq u_{linemn}^{t+1} \leq M(u_{busm}^t + u_{busn}^t) \quad (4)$$

$$t \in \Omega_{t-1}, mn \in \Omega_{line}, m \& n \in \Omega_{bus}$$

where,  $\Omega_{t-1} = 1, \dots, T-1$  is the set of time.

3. For any bus that is not connected with a BSU, if all lines connected with this bus are de-energized, then this bus is de-energized. In other words, bus connected to a BSU will receive power from the BSU; otherwise, at least one of lines connected to this bus has to provide power to energize the bus.

$$-M \sum_{mn \in \Omega_{line-m}} u_{linemn}^t \leq u_{busm}^t \leq M \sum_{mn \in \Omega_{line-m}} u_{linemn}^t \quad (5)$$

$$t \in \Omega_{t-2}, m \in \Omega_{bus} / \Omega_{bus-BSU}$$

where,  $\Omega_{t-2} = 1, \dots, T-2$  is the set of time, and  $\Omega_{bus-BSU}$  is the set of Bus connected with BSU.

4. Once bus or line is energized, it won't be de-energized again.

$$u_{linemn}^t \leq u_{linemn}^{t+1}, \quad t \in \Omega_{t-1}, mn \in \Omega_{line} \quad (6)$$

$$u_{busm}^t \leq u_{busm}^{t+1}, \quad t \in \Omega_{t-1}, m \in \Omega_{bus}$$

5. All the lines' initiate states are de-energized; all the buses' initiate states are de-energized, except the ones connected with BSU are energized.

$$u_{linemn}^1 = 0, \quad t = 1, mn \in \Omega_{line} \quad (7)$$

$$u_{busm}^1 = 1, \quad t = 1, m \in \Omega_{bus-BSU}$$

$$u_{busm}^1 = 0, \quad t = 1, m \in \Omega_{bus} / \Omega_{bus-BSU}$$

6. Transmission line thermal limit constraint

$$-(1 - u_{linemn}^t)M \leq f_{mn}^t - B_{mn}(\theta_m^t - \theta_n^t) \leq (1 - u_{linemn}^t)M \quad (8)$$

$$-u_{linemn}^t F_{mn} \leq f_{mn}^t \leq u_{linemn}^t F_{mn}$$

$$t \in \Omega_t, m \in \Omega_{bus}, mn \in \Omega_{line}$$

where,  $f_{mn}^t$  is the power flow on line  $mn$  at time  $t$ ,  $\theta_m^t$  is the bus  $m$  voltage angle,  $B_{mn}$  is the susceptance of line  $mn$ ,  $F_{mn}$  is the thermal limit for real power flow on line  $mn$ .

## Application in System Restoration

The proposed formulation of transmission path search can be easily connected with generator start-up sequencing module developed in [4] by adding the following constraints:

1. Non-blackstart Unit (NBSU) will be started after the connected bus being energized.

$$u_{busm}^t \geq u_j^t, \quad m \in \Omega_{bus-NBSU_j}, j \in \Omega_{NBSU}, t \in \Omega_t \quad (9)$$

where,  $\Omega_{bus-NBSU_j}$  is the set of Bus connected with  $j^{th}$  NBSU.

2. The starting time of NBSU equals the time when the bus connected with NBSU being energized.

$$\sum_{i \in \Omega_t} (1 - u_j^t) + 1 = \sum_{i \in \Omega_t} (1 - u_{busm}^t) + 1 \quad (10)$$

$$m \in \Omega_{bus-NBSU_j}, j \in \Omega_{NBSU}$$

3. The time when the bus connected with BSU being energized equals the time when the connected BSU starts providing power ( $T_{ictp}$ ).

$$\sum_{i \in \Omega_t} (1 - u_{busm}^t) + 1 = T_{ictp} \quad (11)$$

$$m \in \Omega_{bus-BSU_i}, i \in \Omega_{BSU}$$

The proposed formulation could be extended to include voltage stability constraints. When integrating the transmission path search algorithm with generation and service restoration, AC power flow can be calculated. Voltage security limit and voltage stability margin constraints then can be incorporated to avoid operational issues. These are possible future directions for our work.

## Simulation Results

The IEEE 39-Bus system [9] is used for illustration of the proposed model and solution methodology, as shown in Fig. 3. The developed MILP-based transmission path search (TPS) method will be integrated with the MILP-based generator starting sequence (GSS) method in [4]. There are 10 generators, 39 buses and 46 lines. The generator and transmission system information is given in Table 4-6. The scenario of a complete shutdown is assumed. Unit G10 is a BSU, while G1 – G9 are NBSUs. Per unit (p.u.) time is used in Table 4-6, and 1 p.u. time

equals to 10 minutes. The restoration actions are checked and updated every 10 minutes. It is assumed that BSU can be started at the beginning of system restoration ( $t=0$ ).

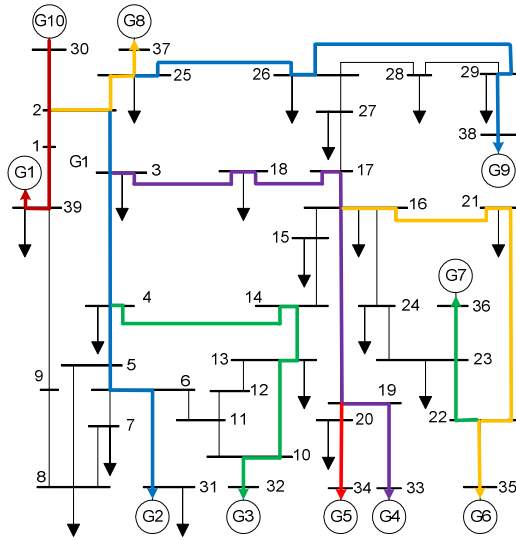


Fig. 3 IEEE 39-Bus System Topology with Optimal Transmission Paths

Table 1 Data of Generator Characteristic

Gen.	$T_{cp}$ (hr)	$Rr$ (MW/hr)	$P_{start}$ (MW)	$P_{max}$ (MW)	Connected Bus
G1	0:30	215	6	573	39
G2	0:30	246	8	650	31
G3	0:30	236	7	632	32
G4	0:30	198	5	508	33
G5	0:30	244	8	650	34
G6	0:30	214	6	560	35
G7	0:30	210	6	540	36
G8	0:30	346	13	830	37
G9	0:30	384	15	1000	38
G10	0:10	162	0	250	30

Table 2 Data of Transmission System

Line	From Bus	To Bus	Line	From Bus	To Bus	Line	From Bus	To Bus
1	30	2	17	5	8	33	16	15
2	2	1	18	8	9	34	15	14
3	1	39	19	6	31	35	14	13
4	39	9	20	6	11	36	24	16
5	37	25	21	12	11	37	24	23
6	25	2	22	12	13	38	16	21
7	5	26	23	11	10	39	21	22
8	2	3	24	10	32	40	35	22
9	18	3	25	13	10	41	19	20
10	18	17	26	26	27	42	20	34
11	3	4	27	26	29	43	16	19
12	4	14	28	26	28	44	19	33
13	4	5	29	28	29	45	22	23
14	5	6	30	29	38	46	23	36
15	6	7	31	27	17			
16	7	8	32	17	16			

The optimal starting times for all generating units are calculated considering different combinations of

optimization modules. First case is to use MILP-based GSS and Correlation Matrix (CB)-based TPS in [4]. After GSS provides the starting sequence of all generating units, CB-based TPS will be used to find the shortest transmission path for sending the cranking power to start up NBSU. Generator start-up sequence and transmission path search are obtained in sequence. Second case is to MILP-based GSS and MILP-based TPS developed in this paper. In this way, GSS and TPS are coupled together to achieve an optimal solution in the same time. The results of all generator start-up times (p.u.), total restoration time (p.u.) and total generation capability (MWh) are shown in Table 3. The comparison of system generation capability curves considering different optimization modules is shown in Fig. 4.

Table 3 Comparison of Generator Start-up Time Considering Different Optimization Modules

	Consider GSS and CB-based TPS	Consider GSS and MILP-based TPS
$t_{1start}$	5	4
$t_{2start}$	7	7
$t_{3start}$	9	8
$t_{4start}$	9	8
$t_{5start}$	10	9
$t_{6start}$	10	9
$t_{7start}$	11	9
$t_{8start}$	5	4
$t_{9start}$	8	8
$t_{10start}$	0	0
Total Restoration Time	30	28
Total Generation Capability	11,836	12,632

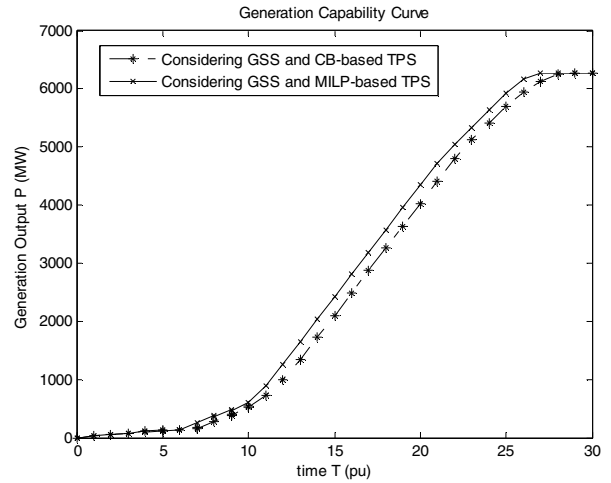


Fig. 4 Comparison of system generation capability curves

It can be observed that generator start-up sequence and time are different using different optimization modules. Considering GSS and TPS together can bring less restoration time and greater generation capability. It demonstrates the advantage of developed MILP-based TPS method to integrate with MILP-based GSS method.

Moreover, the accurate solution can only be achieved by integrating both modules, as developed in this paper.

The energized time of each bus and line are shown in Table 4-5. The optimal transmission path is shown in Fig. 2, and the optimal restoration actions are listed in Table 6. Compared with results in [4], generator start-up time and transmission energization sequence are different. It can be observed that a faster restoration is achieved using the proposed path search method.

Table 4 Energized Time of All Buses

Bus	1	2	3	4	5	6	7	8	9	10
$T_{start}$	4	3	4	5	6	7	9	7	8	8
Bus	11	12	13	14	15	16	17	18	19	20
$T_{start}$	8	8	7	6	7	7	6	5	8	9
Bus	21	22	23	24	25	26	27	28	29	30
$T_{start}$	8	9	9	8	4	7	7	8	8	2
Bus	31	32	33	34	35	36	37	38	39	
$T_{start}$	8	9	9	10	10	10	5	9	5	

Table 5 Energized Time of All Lines

Line	1	2	3	4	5	6	7	8	9	10	11	12
$T_{start}$	3	4	5	26	5	4	7	4	5	6	5	6
Line	13	14	15	16	17	18	19	20	21	22	23	24
$T_{start}$	6	7	9	9	7	8	8	8	9	8	9	9
Line	25	26	27	28	29	30	31	32	33	34	35	36
$T_{start}$	8	9	8	8	9	9	7	7	9	7	7	8
Line	37	38	39	40	41	42	43	44	45	46		
$T_{start}$	9	8	9	10	9	10	8	9	9	10		

Table 6 Restoration Actions

Time	Start Gen	Energize Bus	Energize Line
0	G10	N/A	N/A
1	N/A	N/A	N/A
2	N/A	30	N/A
3	N/A	2	1
4	N/A	1,3,25	2,6,8
5	N/A	4,18,37,39	3,5,9,11
6	G1,G8	5,14,17	10,12,13
7	N/A	6,8,13,15,16,26,27	7,14,17,31,32,34,35
8	N/A	10,11,12,21,24,28,29,31	18,19,20,22,25,27,28,36,38,43
9	G2	7,20,22,23,32,33,38	15,16,24,30,37,39,41,44
10	G3,G4, G9	34,35,36	40,42,46
11	G5,G6,G7	N/A	N/A

## Conclusions

This paper proposed a new method to find optimal transmission path search based on the logical constraints of transmission energization. The MILP formulation enables the integration with other MILP-based restoration modules. Simulation results based on IEEE 39-bus test case demonstrate that the developed model is able to provide the optimal energization time of buses/lines considering generator-starting sequence. As a result, the restoration duration is significantly reduced. In the future

work, the control of sustained overvoltage from energizing lightly loaded transmission line and switched transients from energizing large segments of transmission system will be included in the model to avoid dynamic issues [10].

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